IPP-QM-13: Quantum logic

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MT25

The course

1. Basic quantum formalism
2. Density operators and entanglement
3. Decoherence
4. The measurement problem
5. Dynamical collapse theories
6. Bohmian mechanics
7. Everettian structure
8. Everettian probability
9. EPR and Bell's theorem
10. The Bell-CHSH inequalities and possible responses
11. Contextuality
12. The PBR theorem
13. Quantum logic
14. QBism
15. Pragmatism and relational quantum mechanics
16. Wavefunction realism

Today

Introducing quantum logic

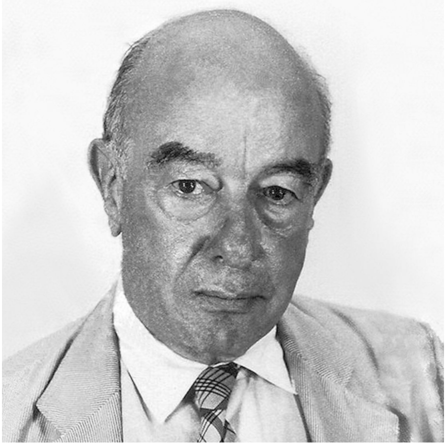
Formalism of quantum logic

Putnam on quantum logic and the measurement problem

Quantum logic and quantum interpretations

Where things stand

Today



W. V. O. Quine (1908–2000)

The Quinean web

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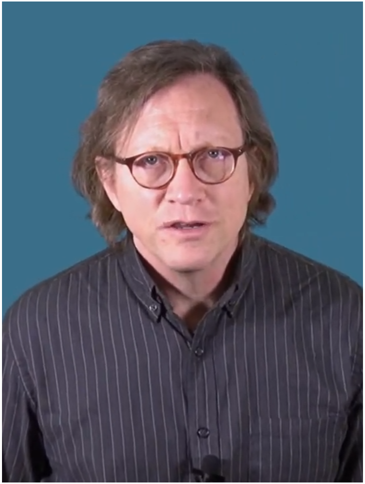
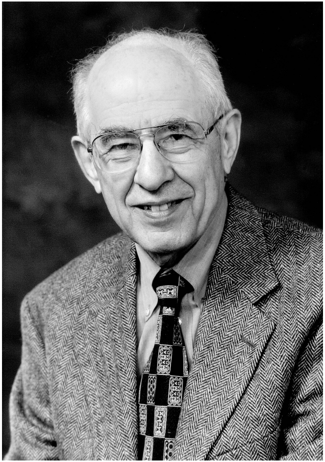


Tim Maudlin (1958–)

The great pantheon of Temptation

Quine here takes his place in the great pantheon of Temptation: the apple hangs glistening before us. Kepler, Einstein, Darwin ... all one has to do to become the next entry on this list is show how a revision in logic could substantially simplify an empirical theory. He even points to the theory: quantum mechanics. Who could resist? ... Fill in the blanks and win immortality! It's an offer you can't refuse. And thereby hangs our tale. (Maudlin, p. 158, 2007)



Hilary Putnam (1926–2016)

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There's now an almost universal consensus that, even if it were otherwise justified, a move to quantum logic would *not* resolve the puzzles of quantum mechanics.

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- In this sense, the first claim is uncontroversial.

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1. Motivating a revision of logic requires not only motivating the introduction of some non-classical connectives. An advocate of a revision of logic must also show why these connectives do not merely sit alongside the classical connectives, but actually *replace* them.
2. A Quinean point: empirical considerations alone cannot force us to revise our logic: a distinctly *philosophical* component will be needed in order to justify whether a revision of logic, as opposed to a revision somewhere else in our network of beliefs, might be desirable.

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This claim is by now recognised to be highly implausible, as we’ll see.

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- But, of course, this is a more general question than anything to do with quantum mechanics *per se*.
- If quantum logic provides us with an intelligible global alternative to classical logic, then the case for logic being empirical might be strengthened.
- However, comprehensive assessment of the question of whether empirical considerations might prompt us to revise our logic will depend less on the details of the physics and more on the largely conceptual question of whether the notion of logical consequence is *a priori* or is an abstraction from what appear to be valid inferences in our language.

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- His claim (2)—that quantum logic is the 'one true logic'—is much more controversial, and must be argued for.
- His claim (3)—that quantum logic itself solves the measurement problem—is also highly questionable, as we'll see.

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- That is, one can define explicitly some non-classical connectives for a certain special class of propositions, relating to idealised quantum mechanical tests.
- These connectives will arguably be well suited for the limited subject matter at hand.
- If as a result one obtains a logical system satisfying certain formal requirements, we shall say that one has introduced a local non-classical logic.

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- On the basis of the above, the law of excluded middle $p \vee \neg p$ breaks down.
- From this, one can set up an intuitionistic logical system (for more on which in general see Sider (2010, ch. 3)).

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Note that quantum logic lacks a material conditional! See Dalla Chiara & Giuntini (2002, §3).

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- $\hat{p} \wedge \hat{q}$ is the intersection of $\hat{p}$ and $\hat{q}$; and
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(Although in previous lectures I was fastidious about not dropping hats on operators, in the remainder of this lecture I will actually drop hats.)

Differences with classical logic: failure of distributivity

The most notable difference between quantum logic and classical logic is the failure of the propositional distributive law:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r).$$

To illustrate why this law fails, let's look at an example.

Illustration of failure of distributivity

Consider a particle moving along a 1D line, and let

- $p =$ "the particle has momentum in the interval $[0, +1/6]$."
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So there are no states that can support either proposition, and $(p \wedge q) \vee (p \wedge r)$ is false; hence, propositional distributivity fails.

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These axioms yield a system of axiomatic proofs for quantum logic (for more on axiomatic proofs, see (Sider 2010)). Alternatively, one can build systems or natural deduction, tableaux, etc.

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- But, as with Putnam (1968), we might seek to move beyond an operational understanding of the approach, and regard quantum logic as bearing on the measurement problem itself.
- Let's explore how Putnam's argument proceeds, and where it goes wrong.

Putnam and the paradoxes of quantum mechanics

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- This last step should already be setting alarm bells ringing...!

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- Putnam (1968, pp. 184–5) concludes that the system possesses values for all observables. (???)
- He then interprets measurements as simply revealing those pre-existing values, thus proposing that the measurement problem of quantum mechanics is solved by a move to quantum logic.

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- (Subtleties to do with decoherence; I'll come back to them.)

Today

Introducing quantum logic

Formalism of quantum logic

Putnam on quantum logic and the measurement problem

Quantum logic and quantum interpretations

Where things stand

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In general, we'll see that the significance of quantum logic will depend upon the interpretation of quantum mechanics which one favours.

The revision of logic

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- And in any case, as we'll see, different interpretations of quantum mechanics will have different things to say on this matter.

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- The resulting procedure is certainly different from that in any classical framework, but there seems to be little need to revise anything but our algorithmic procedures for predicting experimental results.
- So it is not clear why this logic should be even a candidate for a revised global logic.

Quantum mechanics and Bohrian positions

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- Clearly—to repeat—more than empirical considerations are needed in order to mount a case for the revision of logic at the global level.

Quantum logic in realist approaches to QM

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These approaches should give us some resources to explain why classical logic is effective in certain domains of a world in which quantum mechanics is true.

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- At the level of ontic states, configuration-space properties obey classical logic no less than phase-space properties in classical physics.
- Indeed, Bohmian mechanics can be viewed as a theory that is entirely classical at the level of kinematics (particles moving in space and time—see Albert (1992)).
- Thus, the way in which Bohmian mechanics explains the effectivenesss of classical logic at the macroscopic level is that it is already the logic that is operative at the hidden ('untestable') level of the particles.

Bacciagaluppi on quantum logic and Bohmian mechanics

> *Thus, if one takes the pilot-wave approach to quantum mechanics, although quantum logic may be introduced as a local logic at the level of experimental propositions, it cannot be taken as the basis for justifying the everyday use of classical logic, and thus cannot aspire to replace classical logic as the 'true' logic.* (Bacciagaluppi 2013, p. 31)

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> *Thus, also in spontaneous collapse theories (as in pilot-wave theory), the quantum connectives do not provide the basis for the effectiveness of the classical connectives. There is no story explaining that the cat is dead or alive classically because it is dead or alive quantum logically. The cat is first fleetingly (if at all) dead or alive quantum logically, then the dynamics intervenes and ensures the cat is dead or alive classically. Either a hit on the dead component takes place or one on the alive component does.* (Bacciagaluppi 2013, p. 34)

Decoherence and quantum logic in Bohmian mechanics and GRW

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- But of course, in both cases, one in fact still has a quantum state, and so the 'Everett in denial' charge.
- Perhaps the right way to make sense of the emergence of classical logic in both cases, then, is instead via decoherence, *à la* (e.g.) Fortin & Vanni (2014).

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- Under these circumstances (in logic notation), if I have $p \vee q$, I can indeed infer either p or q (inclusive 'or')—so, emergence of the classical 'or' from the quantum 'or'!
- But I still have an improper mixture! So (pace Putnam even here) the measurement problem is not solved.

The Everett interpretation

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- In Everett, the challenge of explaining how classical logic ‘emerges’ from quantum logic is met by decoherence.
- To repeat: decoherence can play this kind of explanatory role in Bohmian mechanics and in dynamical collapse theories too!

Bacciagaluppi on the Everett interpretation

Thus, while the structure of the intrinsic properties of physical systems supports a non-distributive logic at the fundamental level (even in the individual worlds), one can claim that, unlike the case of pilot-wave theory or spontaneous collapse, the perspectival element characteristic of the Everett interpretation introduces a genuine emergence of the classical connectives from the quantum connectives. In this sense, it is only the Everett interpretation, among the major approaches to quantum mechanics, that is compatible with a revision of logic. (Bacciagaluppi 2013, p. 36)

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Questions: What kind of 'perspectivalism' does Bacciagaluppi have in mind here? And does the last claim follow, given what I said previously about decoherence and Bohmian mechanics/GRW?

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Maudlin's final take

The horse of quantum logic has been so thrashed, whipped and pummeled, and is so thoroughly deceased that [...] the question is not whether the horse will rise again, it is: how in the world did this horse get here in the first place? The tale of quantum logic is not the tale of a promising idea gone bad, it is rather the tale of the unrelenting pursuit of a bad idea. [...] Many, many philosophers and physicists have become convinced that a change of logic (and most dramatically, the rejection of classical logic) will somehow help in understanding quantum theory, or is somehow suggested or forced on us by quantum theory. But quantum logic, even through its many incarnations and variations, both in technical form and in interpretation, has never yielded the goods. (Maudlin 2007, pp. 184–5)

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- It is far from evident that quantum logic is a logic, in the sense of describing a process of reasoning (remember, it doesn't have a material conditional!)...
- ...as opposed to a particularly convenient language with which to summarise the *operational* results of quantum mechanical experiments.

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2. Demonstrated that, *pace* Putnam (1968), quantum logic does *not* solve the measurement problem.
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Next time: QBism.

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